Exercise 77

Repeating Decimal Express each repeating decimal as a fraction. (See the margin note on page 3.)

(a) $0.\bar{7}$ **(b)** $0.2\bar{8}$ **(c)** $0.\overline{57}$

Solution

Part (a)

Let $x = 0.777\cdots$. Then

$$\begin{aligned} 100x &= 77.777\cdots \\ 10x &= 7.777\cdots . \end{aligned}$$

Subtract the respective sides of these equations.

$$90x = 70$$

Solve for x.

$$x = \frac{70}{90} = \frac{7}{9}$$

Therefore,

$$0.\bar{7} = \frac{7}{9}.$$

Part (b)

Let $x = 0.2888\cdots$. Then

$$\begin{aligned} 100x &= 28.888\cdots \\ 10x &= 2.888\cdots . \end{aligned}$$

Subtract the respective sides of these equations.

$$90x = 26$$

Solve for x.

$$x = \frac{26}{90} = \frac{13}{45}$$

Therefore,

$$0.2\bar{8} = \frac{13}{45}.$$

Part (c)

Let $x = 0.575757\cdots$. Then

$$10000x = 5757.5757\cdots$$
$$100x = 57.5757\cdots.$$

Subtract the respective sides of these equations.

$$9900x = 5700$$

Solve for x.

$$x = \frac{5700}{9900} = \frac{19}{33}$$

Therefore,

$$0.\overline{57} = \frac{19}{33}.$$